

tion energy;  $F$ , function defined in Eqs. (12) and (13);  $f$ , probability density function;  $g$ , function in Eqs. (14) and (15);  $k$ , reaction rate;  $Q$ , heat of reaction;  $R$ , gas constant;  $T$ , temperature;  $t$ , time;  $w_j$ , Wiener processes;  $x$ , dimensionless concentration;  $z$ , coefficient of the exponential in Eq. (1);  $\alpha$ , heat-transfer coefficient;  $\beta$ , mass influx coefficient of reagent;  $\gamma$ ,  $\delta$ ,  $\mu$ ,  $\sigma$ , parameters defined in Eq. (5);  $\epsilon$ , small parameter in Eq. (1);  $\theta$ , dimensionless temperature;  $\tau$ , dimensionless time;  $\phi$ , probability density function.

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#### RADIATION HEAT TRANSFER IN TWO-PHASE MEDIA

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The state of the art of approximate and numerical methods of the theory of radiation heat transfer is analyzed. The principles for producing engineering methods of computing the radiation heat-transfer characteristics in power plants are examined.

Investigations of radiation heat transfer in two-phase media play an important part in many areas of physics and modern engineering. Their value has recently grown considerably in connection with the rapid development of new techniques and intensification of technological processes associated with the significant increase in the power of energy installations. This results in a need for a more correct solution of radiation heat-transfer problems. The development of methods of the theory of radiation heat transfer should, in our opinion, proceed in two directions, the development of well-founded approximate methods and effective numerical algorithms. If the first methods are needed to carry out correct estimates of the radiation heat-transfer phenomena, to study complex thermophysical processes, then the second is for the production of effective engineering methods and to give a foundation for the approximate methods being developed. Taking into account such important physical phenomena for radiation heat transfer as multiple scattering, selectivity of absorption, thermodynamic nonequilibrium, polydispersity, inhomogeneity and geometry of the emitting volume, etc., is still performed insufficiently correctly in the literature. Even more so, if we speak of a one-time accounting of these phenomena. The solution of such questions would approximate physical models selected to real objects. Moreover, it is necessary to give a physically rigorous foundation of the effective quantities used in radiation heat-transfer practice (effective temperature of the working volume, effective emissivity of the heat carrier, coefficient of thermal efficiency of the screens, etc.). It should be noted that the solution of such problems is also of extreme importance for plasma physics, problems

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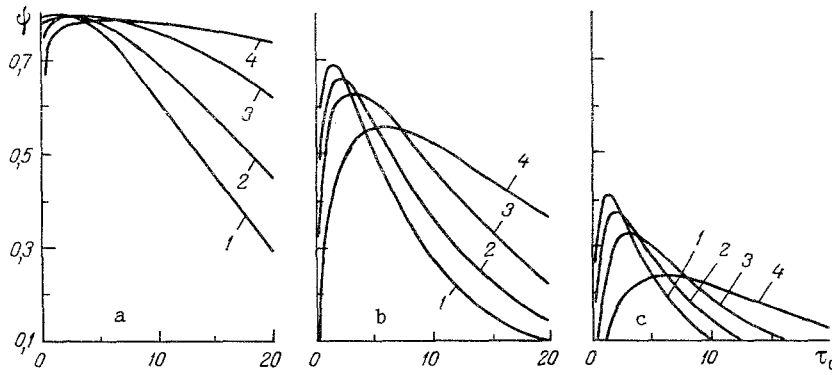


Fig. 1. Dependence of the coefficient of thermal efficiency of screens on the optical thickness for a Schlichting temperature profile ( $\varepsilon_w = 0.8$ ,  $r_w = 0.2$ ): 1)  $Sc = 0$ ; 2) 0.5; 3) 0.8; 4) 0.95; a)  $T_m/T_w = 3$ ,  $\lambda T_m = 5 \cdot 10^{-3}$  mK; b)  $T_m/T_w = 2$ ,  $\lambda T_m = 5 \cdot 10^{-3}$  mK; c)  $T_m/T_w = 2$ ,  $\lambda T_m = 15 \cdot 10^{-3}$  mK.

of the energy regime of earth's atmosphere, for the solution of many questions of modern chemical technology, metallurgy, etc.

The working body in modern power plants is usually a two-phase medium that is a mixture of gaseous combustion products ( $CO_2$ ,  $H_2O$ ,  $CO$ , and other molecular gases) and condensed phase products (soot particles, finely and coarsely dispersed coal, ash) [1]. The study of the regularities of radiation propagation in nonisothermal two-phase media with emitting and reflecting boundary surfaces is fraught with great mathematical difficulties. This requires reliance upon the results of the modern theory of radiation transport [1-6].

The method proposed in [7] can be extracted from the approximate methods of computing the radiation of two-phase media. Its crux is the preliminary approximate determination of the source function and the subsequent integration of the transport equation. Thus, the emissivity of a two-phase layer is determined by the expression [7]:

$$\varepsilon_0 = \frac{1}{B} \int_0^1 I_{out}(\mu) d\mu = \frac{1 - \exp(-k_0 t_0)}{1 + (1 - \varepsilon_\infty) \exp(-k_0 t_0)} \varepsilon_\infty,$$

where

$$B = \frac{\kappa_g B(T_g) + \kappa_{part} B(T_{part})}{\kappa_g + \kappa_{part}}; \quad \varepsilon_\infty = \frac{2}{1 + \delta}; \quad t_0 = \delta^2 (1 - Sc) \tau_0; \quad (1)$$

$$k_0 = 2\delta(1 - Sc); \quad \delta = \left(1 + \frac{2\beta Sc}{1 - Sc}\right)^{\frac{1}{2}}; \quad \mu = \cos \theta.$$

The expression (1) takes account of the anisotropy of scattering as well as the thermodynamic nonequilibrium of the medium ( $T_g$  can differ from  $T_{part}$ ). The quantity  $\varepsilon_\infty$  characterizes the emissivity of a two-phase layer of infinitely large optical thickness. The directional emissivity of a layer has the form [7]

$$\varepsilon_0(\mu) = \frac{I_{out}(\mu)}{B} = 1 - \frac{\varepsilon_\infty \beta Sc}{1 + (1 - \varepsilon_\infty) \exp(-k_0 t_0)} \times \left[ \frac{1 - \exp\left(-\frac{t_0}{\mu} - k_0 t_0\right)}{(1 - Sc) \delta^2 + k_0 \mu} + \frac{\exp(-k_0 t_0) - \exp\left(-\frac{t_0}{\mu}\right)}{(1 - Sc) \delta^2 - k_0 \mu} \right]. \quad (2)$$

The relationships (1) and (2) are significantly more exact than the expressions obtained by the methods of Schwartzschild-Schuster and Eddington, the  $P_1$ -th and  $P_3$ -th approximations of the spherical harmonics method, and the moments and discrete ordinates methods [3]. Moreover, they are more exact even than those approximations selected empirically on the basis of numerical solutions of the transport equations (for instance, with the scattering anisotropy taken into account [8, 9]).

The method under consideration can be utilized to compute the radiation heat transfer in nonisothermal two-phase media bounded by emitting and reflecting surfaces. As is shown in

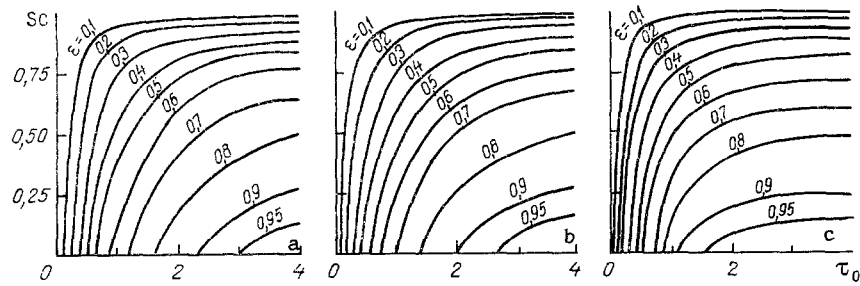


Fig. 2. Nomograms for the computation of the emissivity of a two-phase cylinder for (a)  $\theta = 90^\circ$ ; (b)  $\theta = 60^\circ$ ; (c)  $\theta = 30^\circ$ .

[10], the accuracy of the expressions obtained for a hemispherical and directional emissivity of a layer is sufficiently high. Investigations performed permit the correct introduction of one of the most important concepts of the theory of radiation heat transfer, the effective temperature of a nonisothermal two-phase medium, analyzed in detail in [11]:

$$B(T_{\text{ef}}) = \frac{k}{1 - \exp(-k\tau_0)} \int_0^{\tau_0} B[T(\tau)] \exp(-k\tau) d\tau, \quad k = 2\sqrt{1 - Sc}. \quad (3)$$

By using (3) the coefficient of thermal efficiency of screens is obtained easily [11]

$$\psi = (1 - r_w) \left[ 1 - \frac{b}{N + (1 - r_w) bM} \right], \quad (4)$$

$$N = \frac{\varepsilon_\infty (1 - \exp(-k\tau_0))}{1 - r_w R + (R - r_w) \exp(-k\tau_0)}, \quad R = 1 - \varepsilon_\infty,$$

$$M = \frac{R(1 - r_w R) + (1 - R^2) \exp(-k\tau_0) - (R - r_w) \exp(-2k\tau_0)}{(1 - r_w R)^2 - (R - r_w)^2 \exp(-2k\tau_0)},$$

$$b = \left[ \frac{\exp\left(\frac{c_2}{\lambda T_w}\right) - 1}{1 - \exp(-k\tau_0)} \int_0^{\tau_0} \frac{\exp(-k\tau)}{\exp\left[\frac{c_2}{\lambda T(\tau)}\right] - 1} k d\tau \right]^{-1}.$$

Relationship (4) permits analysis of the thermal efficiency of screens as a function of the optical properties of the boundary surfaces and the furnace medium, the scattering processes, the kind of temperature distribution (Fig. 1).

It is also shown in [7, 8] that the so-called transport approximation [12] can be utilized to compute the radiation of two-phase media with good accuracy, i.e., the radiation scattering index of a volume element is represented by the sum of a certain isotropic part and strongly elongated forward (in the form of a  $\delta$ -function). Analytic expressions for hemispherical and directional emissivities of cylindrical and spherical two-phase media [13] are found when using this approximation and the determination of the source function by the Eddington method [3]. Their accuracy is confirmed by the data of a numerical integration of the radiation transport equation [14]. The results obtained permitted construction, for example, of a nomogram for the computation of the emissivity of a two-phase cylinder as a function of the quantities  $Sc$ ,  $\tau_0$  and  $\theta = \arccos \mu$  (Fig. 2).

Numerical methods [3, 6, 12, etc.] are ordinarily utilized to solve multidimensional radiation transport problems. Different recipes and approximations are used to raise their efficiency. Among the most widespread numerical methods are the S-T method (or the method of determining the reflection and transmission coefficient of a medium) [15], quasidiffusion [16], and also a number of methods based on different difference schemes. Among the latter is the  $S_n$  method (or the Carlson method) [17] and the method of characteristics (or the Vladimirov method [18]). It should be noted that the Monte Carlo method [3, 4] has recently been utilized more and more. However, the sufficiently tedious programs realizable in practice for the computation of radiation characteristics of two-phase media still does not permit execution of mass computations for the selection of optimal operating modes for modern

power plants and, moreover, make radically difficult the solution of complex radiation-convective heat-transfer problems. The program "Raduga" [19], realized in practice and created in the Institute of Applied Mathematics of the Academy of Sciences of the USSR, must be separated out of the programs developed for the computation of the radiation of cylindrical two-phase media. However, because successive approximations is used in this program, the machine time to compute one variation is large for strongly dissipative media or media of large optical thickness (precisely such media more often become the object of investigation in radiation heat-transfer theory). The program mentioned cannot be the basis for creating engineering methods of computing radiation heat transfer, and even more so in those situations when the initial parameters for the computations are known with a definite error.

An effective algorithm is proposed in the monograph [6] for the determination of the spectral and integral radiation characteristics of cylindrical two-phase media with given optical properties of the boundary surfaces. The method of characteristics [18] is the main algorithm as are also finite-difference and iteration methods. Taken as first iteration is the approximate solution [13] which shortens the machine computation time by 1-2 orders.

If the radiation scattering index on a volume element is represented in the form [12, 14]

$$\rho(r, I, I') = 2\beta(r) + 4\pi[1 - 2\beta(r)]\delta(I - I')$$

( $\delta$  is the Dirac function), the radiation transport equation in an infinite cylindrical medium under local thermodynamic equilibrium is [14]

$$\begin{aligned} \sin\theta \left( \cos\varphi \frac{\partial I}{\partial r} - \frac{\sin\varphi}{r} \frac{\partial I}{\partial \varphi} \right) = -[\kappa(r) + 2\beta(r)\sigma(r)] \times \\ \times I(r, \theta, \varphi) + \frac{\beta(r)\sigma(r)}{2\pi} \int_0^{2\pi} d\varphi' \int_0^\pi I(r, \theta', \varphi') \sin\theta' d\theta' + \kappa(r)B(T(r)). \end{aligned} \quad (5)$$

The boundary conditions for this equation should take account of the radiation and selection processes on the side surface

$$\begin{aligned} I(r, \theta, \varphi) \Big|_{r=R, (\ln) < 0} = \varepsilon_w B(T_w) + \frac{2r_w}{\pi^2} \int_{(2\pi)} I(R, \theta', \varphi') \sin\theta' d\Omega', \\ d\Omega = \sin\theta d\theta d\varphi. \end{aligned} \quad (6)$$

The problem can be reformulated if new variables  $x = r \cos \phi$ ,  $y = r \sin \phi$ ,  $\gamma = \cos \theta$  are introduced and the Gauss quadratic formula in  $\gamma$  is utilized

$$\begin{aligned} \sqrt{1 - \gamma_k^2} \frac{\partial I_k}{\partial x} + (\kappa + 2\beta\sigma) I_k(x, y) = S, \\ S = \kappa B + \frac{2\beta\sigma}{\pi} J, \quad J = \sum_{k=1}^N A_k \int_0^\pi I_k(r, \varphi) d\varphi, \\ I_k(x, y) \Big|_{r=R, (\ln) < 0} = \varepsilon_w B(T_w) + \frac{8r_w}{\pi^2} \sum_{k=1}^N A_k \sqrt{1 - \gamma_k^2} \int_0^{\pi/2} I_k(R, \varphi) d\varphi \end{aligned} \quad (7)$$

( $A_k$  and  $\gamma_k$  are values of the weights and abscissas of the Gauss quadrature formula.) Having discretized the problem (7), a recursion relationship for the desired quantities  $I_{i,j,k}$  is easily found

$$I_{i+1,j,k} = I_{i,j,k} q_{i,j,k} + \frac{\Delta x_{i,j}}{2\sqrt{1 - \gamma_k^2}} (S_{i+1} + S_i q_{i,j,k}), \quad (8)$$

where

$$\begin{aligned} S_i = \kappa_i B_i + \frac{2\beta_i \sigma_i}{\pi} J_i; \quad q_{i,j,k} = \exp \left[ - \frac{\kappa_i + 2\beta_i \sigma_i}{\sqrt{1 - \gamma_k^2}} \Delta x_{i,j} \right]; \\ I_{1,j,k} = \varepsilon_w B(T_w) + \frac{4r_w}{\pi^2} \sum_{k=1}^N A_k \sqrt{1 - \gamma_k^2} \sum_{j=1}^L (I_{1,j+1,k} + I_{1,j,k}) |\Delta \varphi_{1,j}|; \end{aligned}$$

$$\Delta x_{i,j} = x_{i,j} - x_{i+1,j}; \quad |\Delta \varphi_{i,j}| = \arcsin \frac{y_{j+1}}{r_i} - \arcsin \frac{y_j}{r_i}.$$

A program "NOTAK" in the language "FORTRAN-IV" was compiled in conformity with (8). The program provides for the computation and print-out of the spectral and integral magnitudes of the radiation intensities and fluxes at any point of the side surface and the coefficient of thermal efficiency of screens as a function of the optical properties of the heat carrier and the boundary surfaces. The quantities mentioned were computed at 32 points for the 1.0-6.0  $\mu\text{m}$  spectrum band. The absorption and scattering coefficient of polydisperse soot, coke, and ash particles were determined by a subprogram according to the Mie theory [6]. The furnace chamber of a powerplant is simulated by a set of cylinders and the edge effects can here be neglected [14]. The computation time for one section over the height of the furnace (or one cylinder) is  $\sim 10$  min.

We used the "NOTAK" program to compute the spectral and integral characteristics of radiation heat transfer in the combustion chamber of a model thermal power plant of TB-240 type. The combustion chamber was simulated by a cylinder ( $R = 500$  cm) and was divided into eight domains ( $\Delta h = 250$  cm) along the height. The water vapor partial pressure and the soot and ash particle concentrations were assumed equally distributed in the combustion chamber ( $P_{\text{H}_2\text{O}} = 0.11$  atm,  $c_s = 4 \cdot 10^{-7}$  g/cm<sup>3</sup>,  $c_a = 2 \cdot 10^{-6}$  g/cm<sup>3</sup>) and the carbon monoxide and coke particle content varied along the height. Presented in Fig. 3, for example, is the spectral distribution of the thermal efficiency coefficient for screens  $\psi_\lambda$  for two sections of the model installation under consideration. The quantity  $\psi_\lambda$  is sharply selective in nature and can vary radically over the height of the installation. The physical conditions in the selected sections differ substantially (in the second section  $T_w = 1000^\circ\text{K}$ ,  $T = 1200$ - $1275^\circ\text{K}$ , and in the fourth  $T_w = 1300^\circ\text{K}$ ,  $T = 1500$ - $1800^\circ\text{K}$ ). As is seen from the figure, the soot particles in the tongue exert the greatest influence on the selectivity of the thermal efficiency coefficient of screens. The maximum values of  $\psi_\lambda$  are found in the spectrum ranges 1.0-2.5  $\mu\text{m}$  and 4.2-4.6  $\mu\text{m}$ .

Therefore, the radiation heat transfer characteristics depend in a substantial manner on the initial spectroscopic data on the gas components (water vapor, carbon monoxide, carbon dioxide) and the condensed phase particles (soot, coal, and ash particles). The second important factor governing the radiation field in the combustion chambers of thermoelectric plants is the temperature distribution in the furnace medium. It depends substantially on the power plant construction itself and on the organization of the fuel ignition process. And finally, the optical properties of the contaminated heat perceiving surface outcrops of the screen heating play an important role. Consequently, the accuracy of the proposed engineering method of computing the radiation heat transfer characteristics is determined mainly by the closeness of the model geometry to the combustion chamber under investigation and by the set of initial parameters on the physicochemical structure of the power plant tongues.

At this time the problem of automating the checking and regulation processes of thermal power plants is acute. Its solution is associated directly with the necessity to investigate the influence of different factors (the size and configuration of the furnace chamber, the construction and composition of the burners, the kind of combustible fuel, the fuel consumption, etc.) on the course of the thermal processes proceeding in power plant chambers, and on the radiation heat transfer characteristics by means of them. This is indeed the basis for constructing contactless methods of checking and automating thermal aggregate operation.

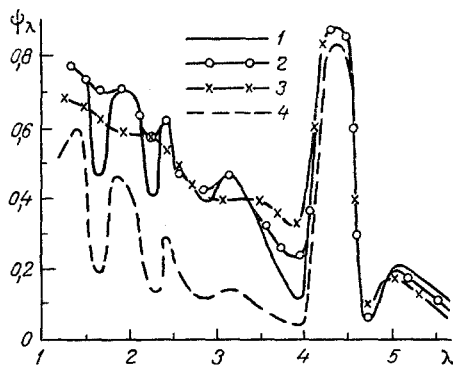


Fig. 3. Spectral dependence of the thermal efficiency coefficient of thermally perceptive surfaces of the combustion chamber of a model installation: 1) pure gas; 2) gas plus soot; 3) gas + ash (second section); 4) pure gas (fourth section).  $\lambda$ ,  $\mu\text{m}$ .

## NOTATION

$I_\lambda(r, \theta, \phi)$ , intensity of radiation of wavelength  $\lambda$  at a point  $r$  and direction  $I = I(\theta, \phi)$ ;  $I_{out}$ , radiation intensity leaving from a medium;  $J$ , radiation intensity averaged with respect to direction;  $S$ , source function;  $B(T_k)$ , Planck radiation intensity at the temperature  $T_k$ ;  $\epsilon$ , emissivity of the medium;  $\psi$ , screen thermal efficiency coefficient;  $\tau_0 = \int_0^{x_0} (\kappa_g + \kappa_{part} + \sigma) dx$ , optical thickness of a layer (optical radius of a cylinder or sphere);  $\kappa_g$  and  $\kappa_{part}$ , gas and particle absorption coefficients, respectively;  $\sigma$ , particle scattering coefficient;  $Sc = \sigma / (\kappa_g + \kappa_{part} + \sigma)$ , Schuster criterion (or the probability of photon survival);  $p(r, \theta, \phi; \theta', \phi')$ , radiation scattering index of a volume element of the medium;  $\beta$ , hemispherical backscatter fraction;  $\epsilon_w$ ,  $r_w$  and  $T_w$ , boundary surface (wall) emissivity, reflection coefficient, and temperature;  $T_m$ , temperature at the middle of the layer;  $\mu = \cos \theta$ .

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